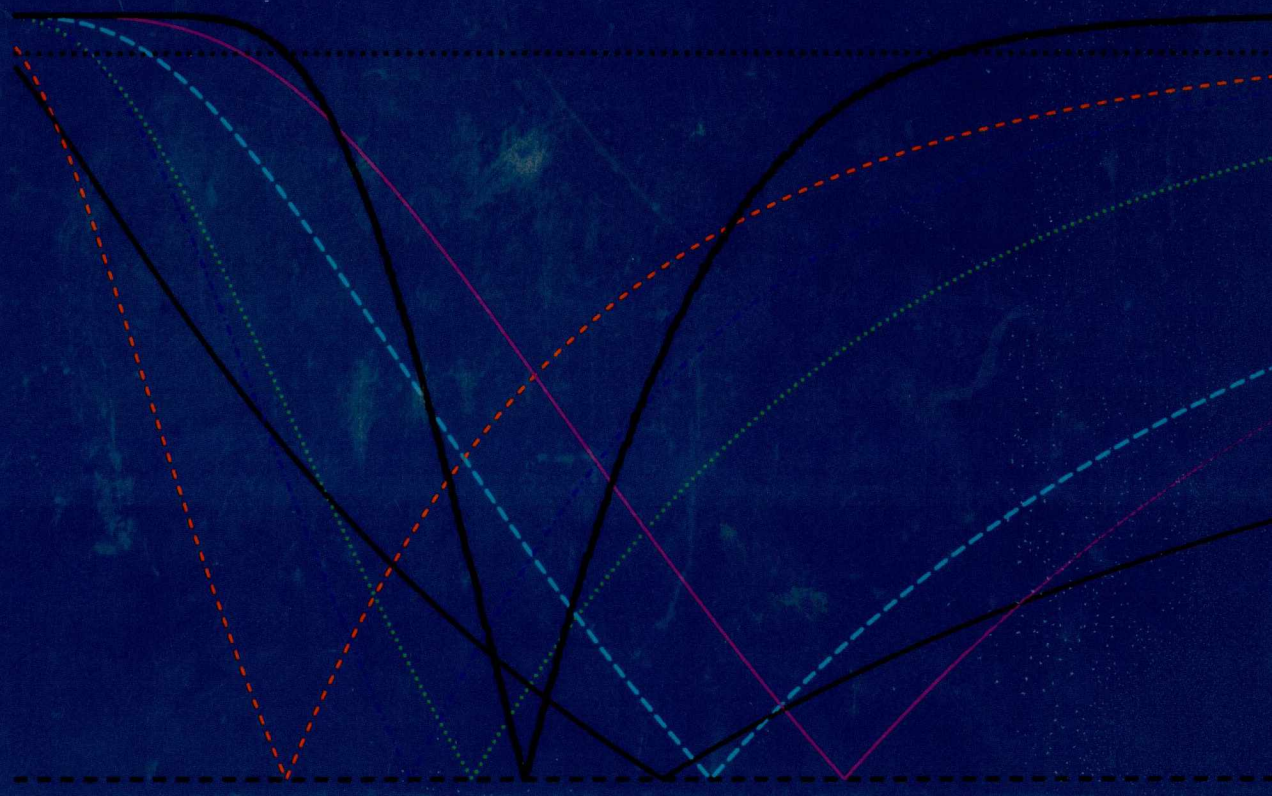


Cambridge Series in Statistical  
and Probabilistic Mathematics

# Confidence, Likelihood, Probability

Statistical Inference with  
Confidence Distributions

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# Contents

*Preface*

*page xiii*

|          |                                                           |           |
|----------|-----------------------------------------------------------|-----------|
| <b>1</b> | <b>Confidence, likelihood, probability: An invitation</b> | <b>1</b>  |
| 1.1      | Introduction                                              | 1         |
| 1.2      | Probability                                               | 4         |
| 1.3      | Inverse probability                                       | 6         |
| 1.4      | Likelihood                                                | 7         |
| 1.5      | Frequentism                                               | 8         |
| 1.6      | Confidence and confidence curves                          | 10        |
| 1.7      | Fiducial probability and confidence                       | 14        |
| 1.8      | Why not go Bayesian?                                      | 16        |
| 1.9      | Notes on the literature                                   | 19        |
| <br>     |                                                           |           |
| <b>2</b> | <b>Inference in parametric models</b>                     | <b>23</b> |
| 2.1      | Introduction                                              | 23        |
| 2.2      | Likelihood methods and first-order large-sample theory    | 24        |
| 2.3      | Sufficiency and the likelihood principle                  | 30        |
| 2.4      | Focus parameters, pivots and profile likelihoods          | 32        |
| 2.5      | Bayesian inference                                        | 40        |
| 2.6      | Related themes and issues                                 | 42        |
| 2.7      | Notes on the literature                                   | 48        |
|          | Exercises                                                 | 50        |
| <br>     |                                                           |           |
| <b>3</b> | <b>Confidence distributions</b>                           | <b>55</b> |
| 3.1      | Introduction                                              | 55        |
| 3.2      | Confidence distributions and statistical inference        | 56        |
| 3.3      | Graphical focus summaries                                 | 65        |
| 3.4      | General likelihood-based recipes                          | 69        |
| 3.5      | Confidence distributions for the linear regression model  | 72        |
| 3.6      | Contingency tables                                        | 78        |
| 3.7      | Testing hypotheses via confidence for alternatives        | 80        |
| 3.8      | Confidence for discrete parameters                        | 83        |

|          |                                                                            |     |
|----------|----------------------------------------------------------------------------|-----|
| 3.9      | Notes on the literature                                                    | 91  |
|          | Exercises                                                                  | 92  |
| <b>4</b> | <b>Further developments for confidence distribution</b>                    | 100 |
| 4.1      | Introduction                                                               | 100 |
| 4.2      | Bounded parameters and bounded confidence                                  | 100 |
| 4.3      | Random and mixed effects models                                            | 107 |
| 4.4      | The Neyman–Scott problem                                                   | 111 |
| 4.5      | Multimodality                                                              | 115 |
| 4.6      | Ratio of two normal means                                                  | 117 |
| 4.7      | Hazard rate models                                                         | 122 |
| 4.8      | Confidence inference for Markov chains                                     | 128 |
| 4.9      | Time series and models with dependence                                     | 133 |
| 4.10     | Bivariate distributions and the average confidence density                 | 138 |
| 4.11     | Deviance intervals versus minimum length intervals                         | 140 |
| 4.12     | Notes on the literature                                                    | 142 |
|          | Exercises                                                                  | 144 |
| <b>5</b> | <b>Invariance, sufficiency and optimality for confidence distributions</b> | 154 |
| 5.1      | Confidence power                                                           | 154 |
| 5.2      | Invariance for confidence distributions                                    | 157 |
| 5.3      | Loss and risk functions for confidence distributions                       | 161 |
| 5.4      | Sufficiency and risk for confidence distributions                          | 165 |
| 5.5      | Uniformly optimal confidence for exponential families                      | 173 |
| 5.6      | Optimality of component confidence distributions                           | 177 |
| 5.7      | Notes on the literature                                                    | 179 |
|          | Exercises                                                                  | 180 |
| <b>6</b> | <b>The fiducial argument</b>                                               | 185 |
| 6.1      | The initial argument                                                       | 185 |
| 6.2      | The controversy                                                            | 188 |
| 6.3      | Paradoxes                                                                  | 191 |
| 6.4      | Fiducial distributions and Bayesian posteriors                             | 193 |
| 6.5      | Coherence by restricting the range: Invariance or irrelevance?             | 194 |
| 6.6      | Generalised fiducial inference                                             | 197 |
| 6.7      | Further remarks                                                            | 200 |
| 6.8      | Notes on the literature                                                    | 201 |
|          | Exercises                                                                  | 202 |
| <b>7</b> | <b>Improved approximations for confidence distributions</b>                | 204 |
| 7.1      | Introduction                                                               | 204 |
| 7.2      | From first-order to second-order approximations                            | 205 |

|           |                                                           |     |
|-----------|-----------------------------------------------------------|-----|
| 7.3       | Pivot tuning                                              | 208 |
| 7.4       | Bartlett corrections for the deviance                     | 210 |
| 7.5       | Median-bias correction                                    | 214 |
| 7.6       | The t-bootstrap and abc-bootstrap method                  | 217 |
| 7.7       | Saddlepoint approximations and the magic formula          | 219 |
| 7.8       | Approximations to the gold standard in two test cases     | 222 |
| 7.9       | Further remarks                                           | 227 |
| 7.10      | Notes on the literature                                   | 228 |
|           | Exercises                                                 | 229 |
| <b>8</b>  | <b>Exponential families and generalised linear models</b> | 233 |
| 8.1       | The exponential family                                    | 233 |
| 8.2       | Applications                                              | 235 |
| 8.3       | A bivariate Poisson model                                 | 241 |
| 8.4       | Generalised linear models                                 | 246 |
| 8.5       | Gamma regression models                                   | 249 |
| 8.6       | Flexible exponential and generalised linear models        | 252 |
| 8.7       | Strauss, Ising, Potts, Gibbs                              | 256 |
| 8.8       | Generalised linear-linear models                          | 260 |
| 8.9       | Notes on the literature                                   | 264 |
|           | Exercises                                                 | 266 |
| <b>9</b>  | <b>Confidence distributions in higher dimensions</b>      | 274 |
| 9.1       | Introduction                                              | 274 |
| 9.2       | Normally distributed data                                 | 275 |
| 9.3       | Confidence curves from deviance functions                 | 278 |
| 9.4       | Potential bias and the marginalisation paradox            | 279 |
| 9.5       | Product confidence curves                                 | 280 |
| 9.6       | Confidence bands for curves                               | 284 |
| 9.7       | Dependencies between confidence curves                    | 291 |
| 9.8       | Notes on the literature                                   | 292 |
|           | Exercises                                                 | 292 |
| <b>10</b> | <b>Likelihoods and confidence likelihoods</b>             | 295 |
| 10.1      | Introduction                                              | 295 |
| 10.2      | The normal conversion                                     | 298 |
| 10.3      | Exact conversion                                          | 301 |
| 10.4      | Likelihoods from prior distributions                      | 302 |
| 10.5      | Likelihoods from confidence intervals                     | 305 |
| 10.6      | Discussion                                                | 311 |
| 10.7      | Notes on the literature                                   | 312 |
|           | Exercises                                                 | 313 |

|           |                                                           |     |
|-----------|-----------------------------------------------------------|-----|
| <b>11</b> | <b>Confidence in non- and semiparametric models</b>       | 317 |
| 11.1      | Introduction                                              | 317 |
| 11.2      | Confidence distributions for distribution functions       | 318 |
| 11.3      | Confidence distributions for quantiles                    | 318 |
| 11.4      | Wilcoxon for location                                     | 324 |
| 11.5      | Empirical likelihood                                      | 325 |
| 11.6      | Notes on the literature                                   | 332 |
|           | Exercises                                                 | 333 |
| <b>12</b> | <b>Predictions and confidence</b>                         | 336 |
| 12.1      | Introduction                                              | 336 |
| 12.2      | The next data point                                       | 337 |
| 12.3      | Comparison with Bayesian prediction                       | 343 |
| 12.4      | Prediction in regression models                           | 346 |
| 12.5      | Time series and kriging                                   | 350 |
| 12.6      | Spatial regression and prediction                         | 353 |
| 12.7      | Notes on the literature                                   | 356 |
|           | Exercises                                                 | 356 |
| <b>13</b> | <b>Meta-analysis and combination of information</b>       | 360 |
| 13.1      | Introduction                                              | 360 |
| 13.2      | Aspects of scientific reporting                           | 363 |
| 13.3      | Confidence distributions in basic meta-analysis           | 364 |
| 13.4      | Meta-analysis for an ensemble of parameter estimates      | 371 |
| 13.5      | Binomial count data                                       | 374 |
| 13.6      | Direct combination of confidence distributions            | 375 |
| 13.7      | Combining confidence likelihoods                          | 376 |
| 13.8      | Notes on the literature                                   | 379 |
|           | Exercises                                                 | 380 |
| <b>14</b> | <b>Applications</b>                                       | 383 |
| 14.1      | Introduction                                              | 383 |
| 14.2      | Golf putting                                              | 384 |
| 14.3      | Bowheads                                                  | 387 |
| 14.4      | Sims and economic prewar development in the United States | 389 |
| 14.5      | Olympic unfairness                                        | 391 |
| 14.6      | Norwegian income                                          | 396 |
| 14.7      | Meta-analysis of two-by-two tables from clinical trials   | 401 |
| 14.8      | Publish (and get cited) or perish                         | 409 |
| 14.9      | Notes on the literature                                   | 412 |
|           | Exercises                                                 | 413 |

|           |                                                            |     |
|-----------|------------------------------------------------------------|-----|
| <b>15</b> | <b>Finale: Summary, and a look into the future</b>         | 418 |
| 15.1      | A brief summary of the book                                | 418 |
| 15.2      | Theories of epistemic probability and evidential reasoning | 423 |
| 15.3      | Why the world need not be Bayesian after all               | 428 |
| 15.4      | Unresolved issues                                          | 430 |
| 15.5      | Finale                                                     | 435 |
|           | <i>Overview of examples and data</i>                       | 437 |
|           | <i>Appendix: Large-sample theory with applications</i>     | 447 |
| A.1       | <i>Convergence in probability</i>                          | 447 |
| A.2       | <i>Convergence in distribution</i>                         | 448 |
| A.3       | <i>Central limit theorems and the delta method</i>         | 449 |
| A.4       | <i>Minimisers of random convex functions</i>               | 452 |
| A.5       | <i>Likelihood inference outside model conditions</i>       | 454 |
| A.6       | <i>Robust parametric inference</i>                         | 458 |
| A.7       | <i>Model selection</i>                                     | 462 |
| A.8       | <i>Notes on the literature</i>                             | 464 |
|           | <i>Exercises</i>                                           | 464 |
|           | <i>References</i>                                          | 471 |
|           | <i>Name index</i>                                          | 489 |
|           | <i>Subject index</i>                                       | 495 |